Enhanced Kauffman Bracket: Talk References

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Knots & Graphs 2017

Notation for (Bicolored) Crossings:

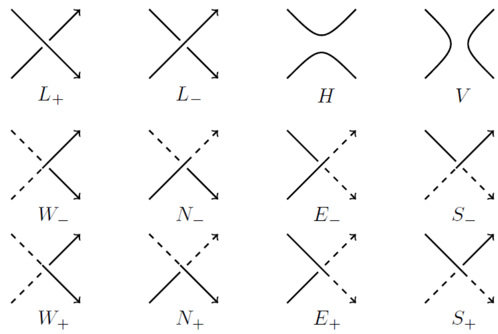


FIGURE 1. Local crossings.

Skein Relations (Eq.(1)):

$$N_+ = a_n H + b_n V \qquad N_- = a'_n H + b'_n V$$

$$S_+ = a_s H + b_s V \qquad S_- = a'_s H + b'_s V$$

$$E_+ = a_e H + b_e V \qquad E_- = a'_e H + b'_e V$$

$$W_+ = a_w H + b_w V \qquad W_- = a'_w H + b'_w V$$

$$< D \sqcup \bigcirc >= d < D >$$

Reidemeister Move Type II Restrictions, Simplified (Eq.(2)):

$$a'_e = a_w^{-1} \qquad a'_w = a_e^{-1} \qquad a'_s = a_s^{-1} \qquad a'_n = a_n^{-1}$$

$$b'_e = b_w^{-1} \qquad b'_w = b_e^{-1} \qquad b'_s = b_s^{-1} \qquad b'_n = b_n^{-1}$$

$$-d = \frac{a_w}{b_w} + \frac{b_w}{a_w} = \frac{a_e}{b_e} + \frac{b_e}{a_e} = \frac{a_s}{b_s} + \frac{b_s}{a_s} = \frac{a_n}{b_n} + \frac{b_n}{a_n}$$

Theorem (Minimal Generating Set of Reidemeister Moves, Polyak) Two equivalent diagrams can be obtained from each other through isotopy and a finite sequence of four oriented Reidemeister moves:

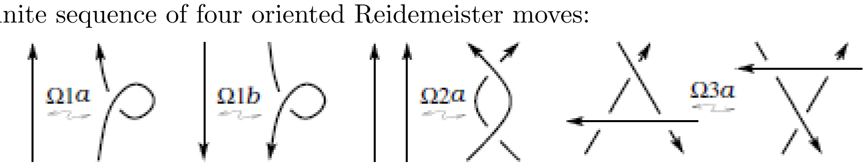
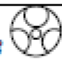
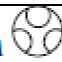
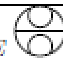
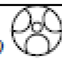
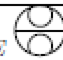


Table Inside Smoothings of Reidemeister III Disks

	$a_1a_2a_3$	$a_1a_2b_3$	$a_1b_2a_3$	$a_1b_2b_3$	$b_1a_2a_3$	$b_1a_2b_3$	$b_1b_2a_3$	$b_1b_2b_3$
L	B 	A 	A 	E 	A 	F 	G 	C 
L'	D 	C 	C 	E 	C 	F 	G 	A 

Reidemeister Moves Type II & III Restrictions (Eq.(3)):

Let $a, b, n, s, w,$ and e be parameters for Eqs.(2) above. Then R-III yields that $n = s$ and we have:

$$a_n = a_s = na \quad b_n = b_s = nb \quad a_w = wa \quad b_w = wb \quad a_e = ea \quad b_e = eb$$
$$a'_n = a'_s = \frac{1}{na} \quad b'_n = b'_s = \frac{1}{nb} \quad a'_w = \frac{1}{ea} \quad b'_w = \frac{1}{eb} \quad a'_e = \frac{1}{wa} \quad b'_e = \frac{1}{wb}$$
$$d = -\frac{a}{b} - \frac{b}{a}$$

Theorem (The Knot/Link Invariant)

Let $F(D) = \left(-\frac{b}{na^2}\right)^{w(D)} < D >$, where $w(D)$ denotes the writhe. Let $\overline{F}(D)$ denote the polynomial obtained from the other coloring of the knot. (The second coloring is obtained from the first by setting $\overline{w} = e$ and $\overline{e} = w$.)

For links, let Λ denote the set of all colorings for the link diagram, and $F(D, \lambda)$ the polynomial from a specific coloring $\lambda \in \Lambda$.

Using Skein relations (1), if equations (3) are satisfied, then $\{F(D), \overline{F}(D)\}$ is a knot invariant.

For a link L, choose one diagram D. Then $F(L) = \{F(D, \lambda) | \lambda \in \Lambda\}$ is a multivalued link invariant.

Example of Choice of Coefficients in $\mathbf{Z_2[t]/(1+t+t^3)}$

$$\begin{cases} a_n = a_s = 1, b_n = b_s = t, a_w = 1 + t^2, b_w = 1, a_e = 1 + t, b_e = t + t^2, \\ a'_n = a'_s = 1, b'_n = b'_s = 1 + t^2, a'_w = t + t^2, b'_w = 1 + t, a'_e = t, b'_e = 1 \\ d = 1 + t + t^2 \end{cases}$$

References

1. Z. Yang, *Enhanced Kauffman Bracket*, arXiv:1702.03391

2. M. Polyak, *Minimal generating sets of Reidemeister moves*, Quantum Topology, vol. 1, Issue 4, (2010), 399C411.